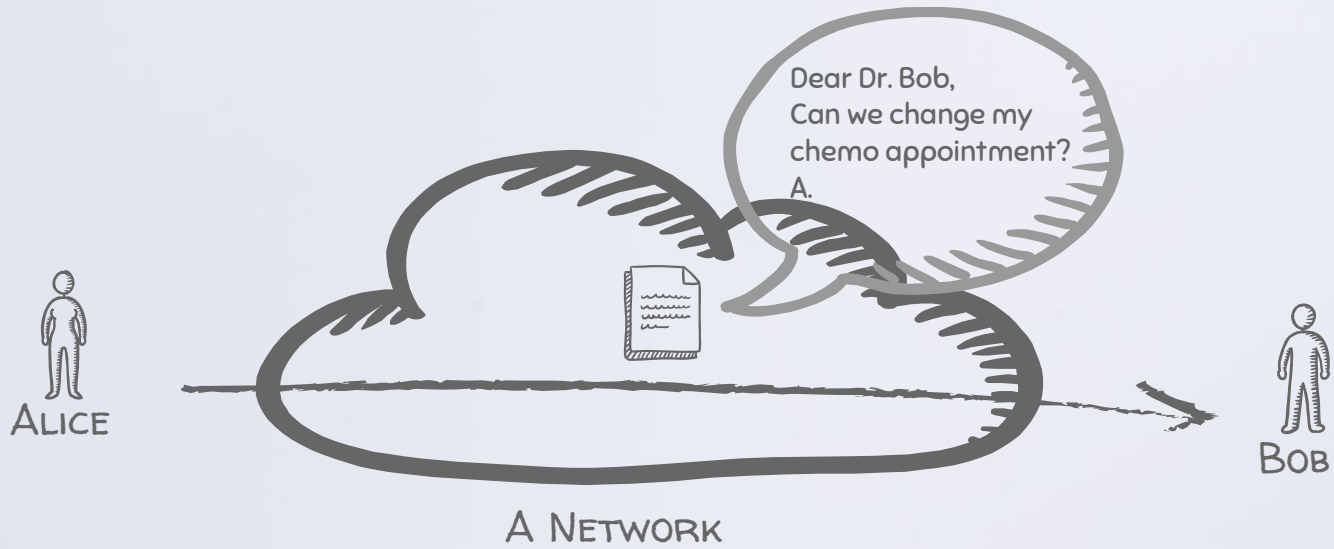


# TRAFFIC ANALYSIS: OR... ENCRYPTION IS NOT ENOUGH

CARMELA TRONCOSO\*  
IMDEA SOFTWARE INSTITUTE  
SUMMER RESEARCH INSTITUTE  
24<sup>TH</sup> JUNE 2016

\*THANKS TO GEORGE DANEZIS FOR SHARING SLIDES

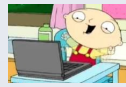
# PRIVACY IN ELECTRONIC COMMUNICATIONS



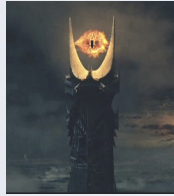
# PRIVACY IN ELECTRONIC COMMUNICATIONS



Your Parents



Your Children



The Boss



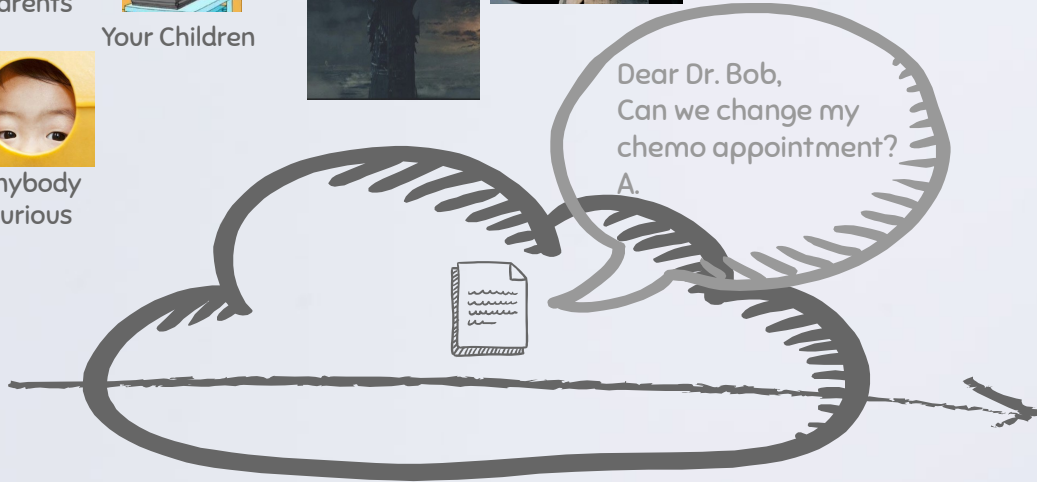
Anybody curious



ISPs



ALICE

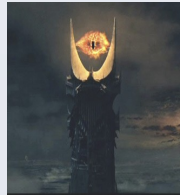


BOB

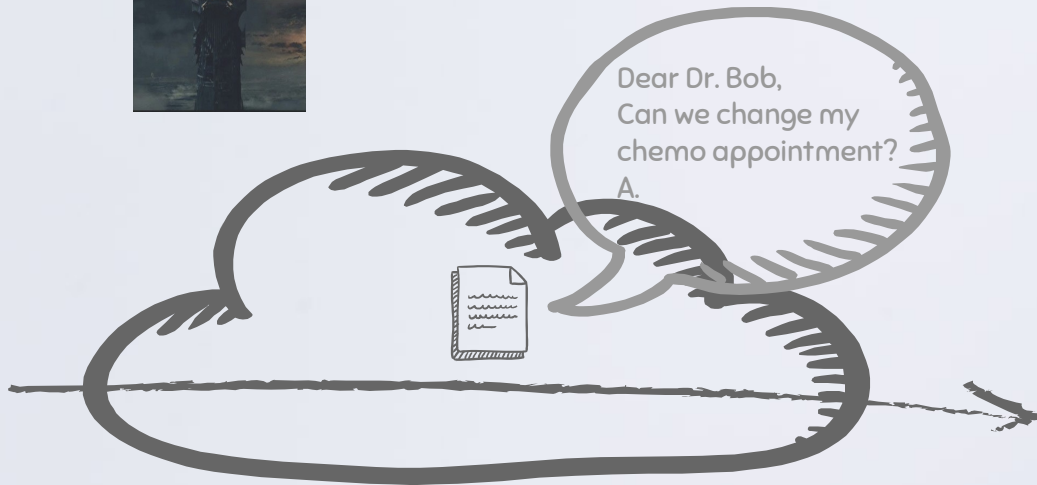
A NETWORK



# BUT WE CAN ENCRYPT! WHAT IS THE PROBLEM?



ALICE

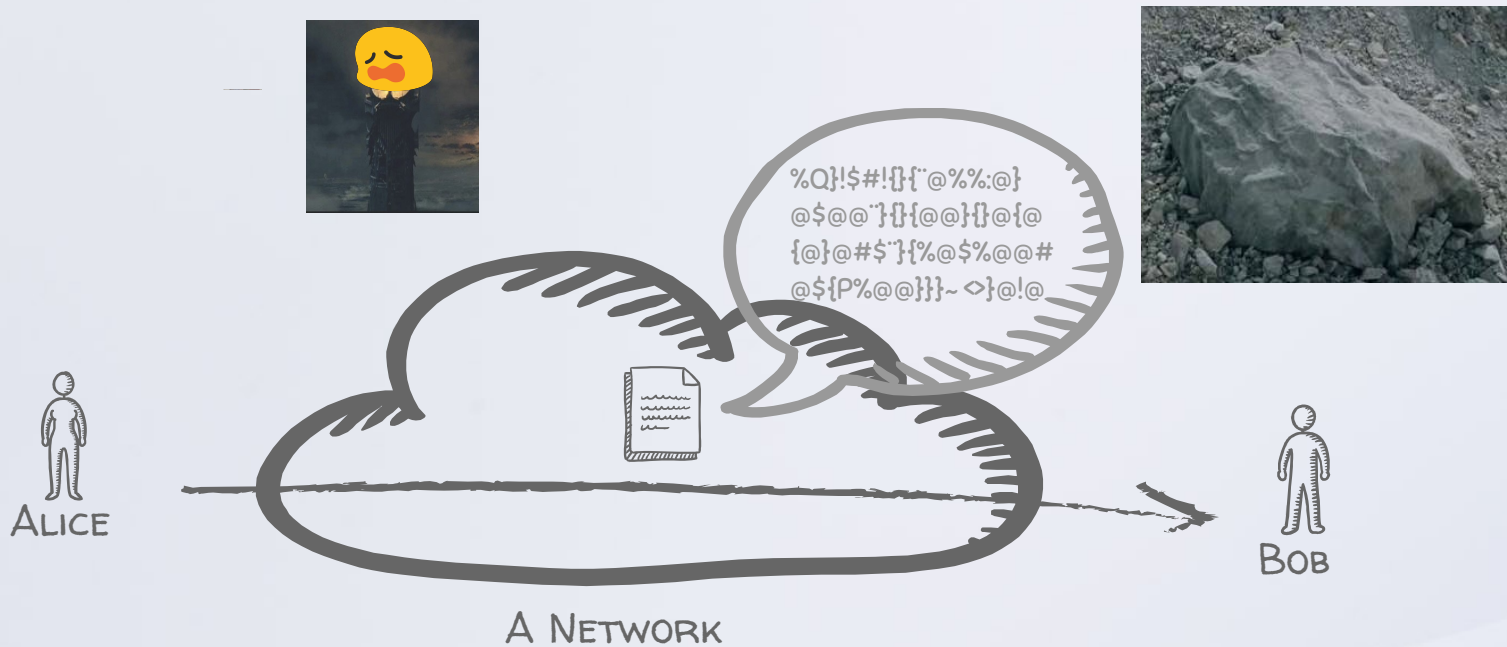


BOB

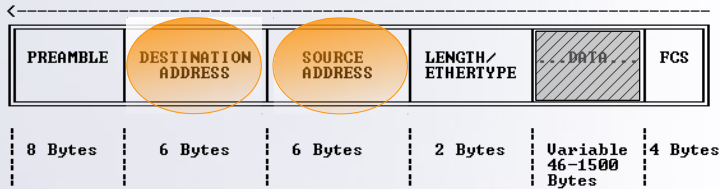
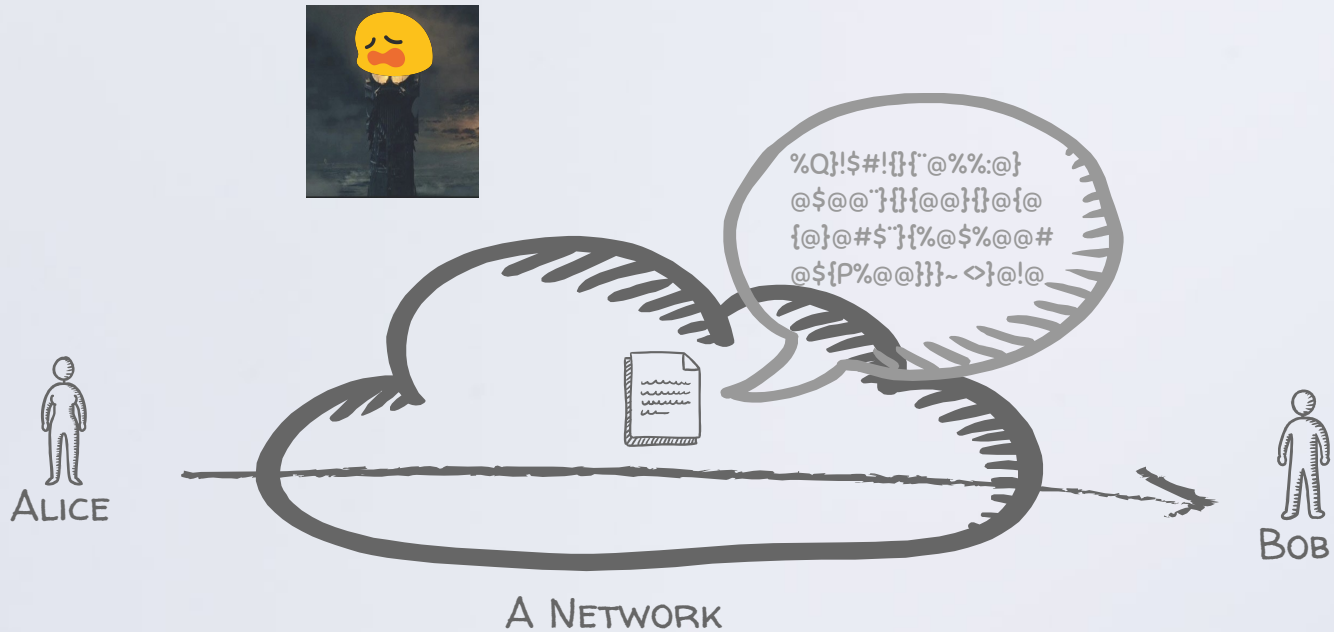
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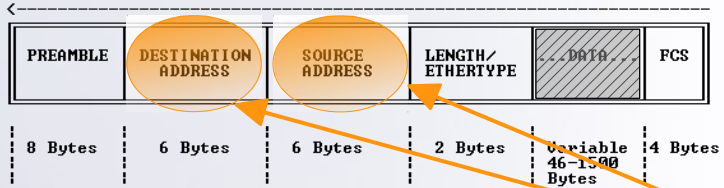
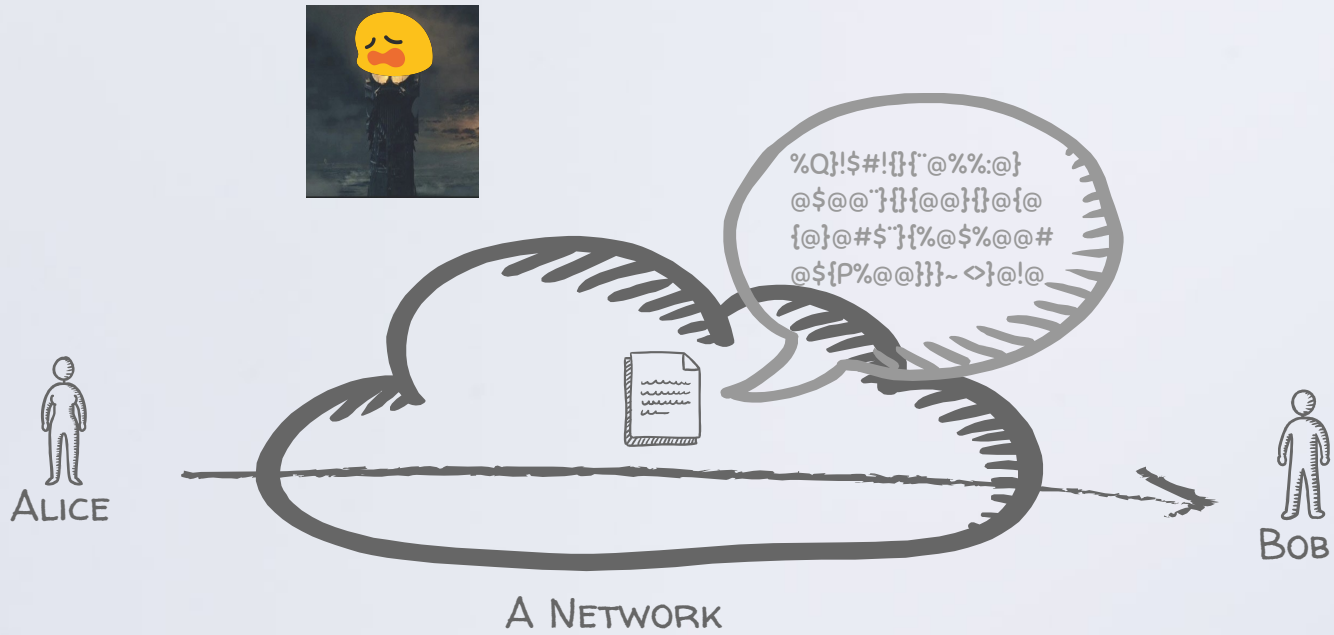


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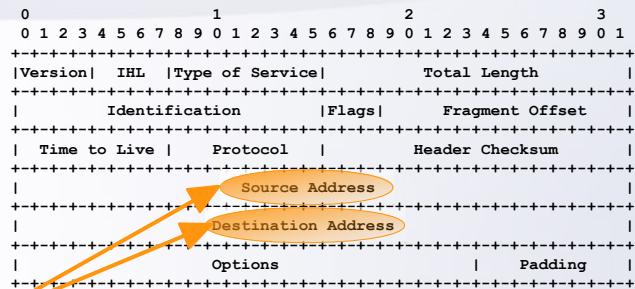


ETHERNET  
(IEEE 802.3, 1997)

# BUT WE CAN ENCRYPT! WHAT IS THE PROBLEM?



ETHERNET (IEEE 802.3, 1997)

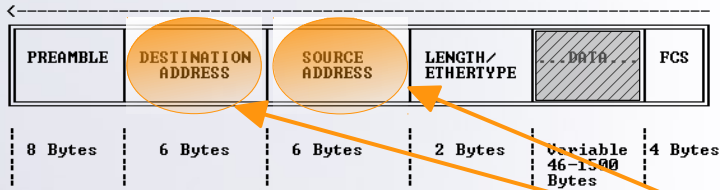
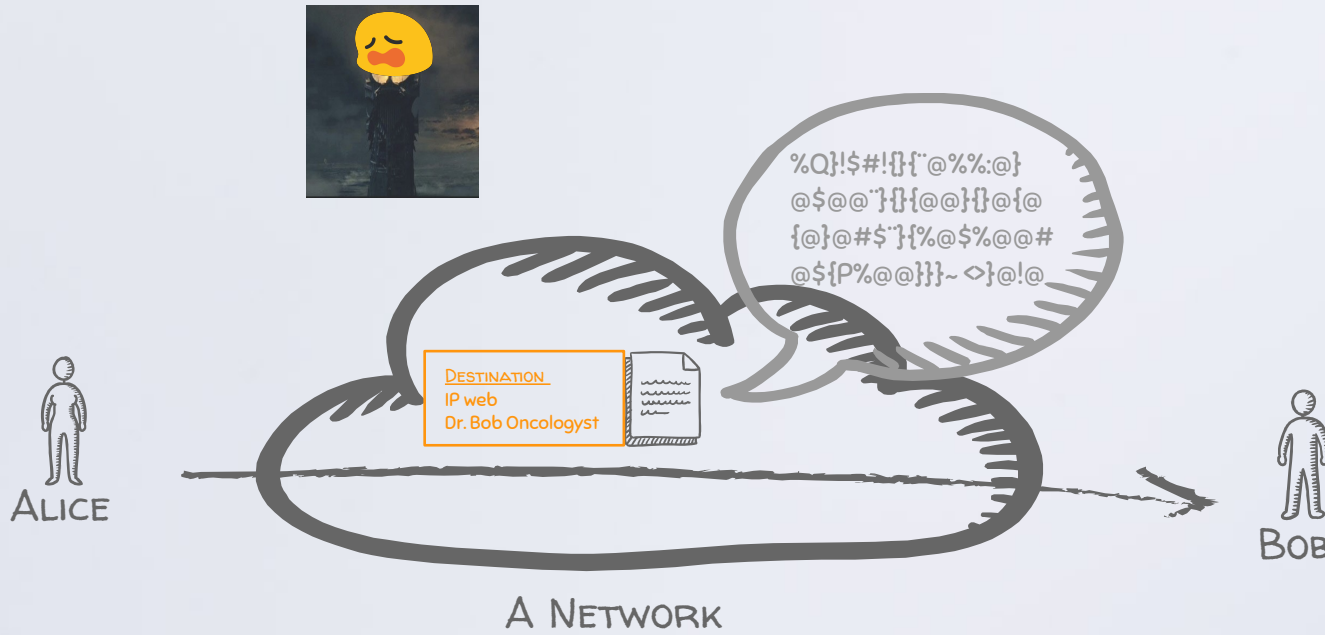


WEAK IDENTIFIER

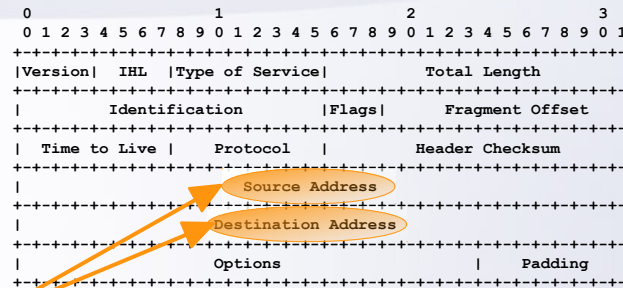
IPV4 HEADER (RFC 791, 1981)

Same for TCP, SMTP, IRC, HTTP, ...

# BUT WE CAN ENCRYPT! WHAT IS THE PROBLEM?



ETHERNET (IEEE 802.3, 1997)



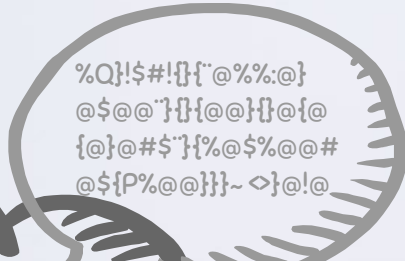
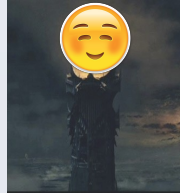
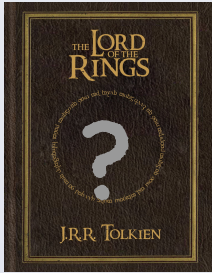
WEAK IDENTIFIER

IPV4 HEADER (RFC 791, 1981)

Same for TCP, SMTP, IRC, HTTP, ...



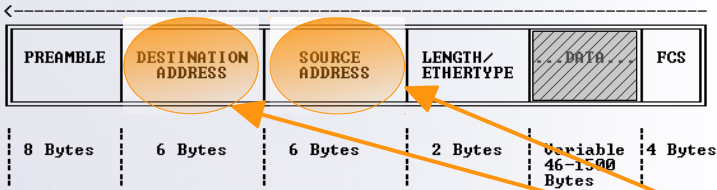
# OMG!! THE PROBLEM IS TRAFFIC ANALYSIS!!



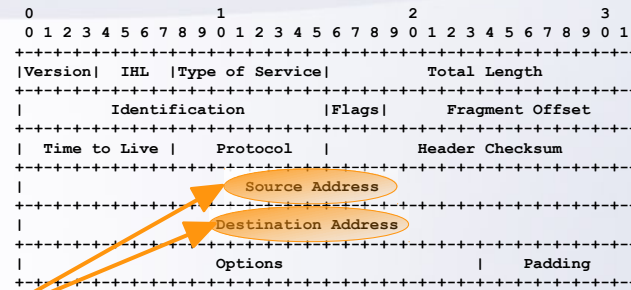
DESTINATION  
IP web  
Dr. Bob Oncologist



A NETWORK



ETHERNET  
(IEEE 802.3, 1997)



WEAK IDENTIFIER

IPV4 HEADER  
(RFC 791, 1981)

Same for TCP, SMTP, IRC, HTTP, ...

# TRAFFIC WHAT?

WIKIPEDIA: traffic analysis is the process of intercepting and examining messages in order to deduce information from patterns in communication

MAKING USE OF "JUST" TRAFFIC DATA OF A COMMUNICATION (AKA METADATA) TO EXTRACT INFORMATION  
(AS OPPOSED TO ANALYZING CONTENT OR PERFORM CRYPTANALYSIS)



Identities of  
communicating parties



Timing, frequency,  
duration



Location



Volume



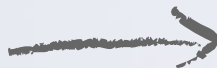
Device

## MILITARY ROOTS

- M. Herman: "These non-textual techniques can establish **TARGETS' LOCATIONS**, order-of-battle and **MOVEMENT**. Even when messages are not being deciphered, traffic analysis of the target's Command, Control, Communications and intelligence system and its patterns of behavior provides indications of his **INTENTIONS** and **STATES OF MIND**"

- **WWI**: British troops finding German boats.

- **WWII**: assessing size of German Air Force, fingerprinting of transmitters or operators (localization of troops).



## NOWADAYS

- Diffie&Landau: "Traffic analysis, not cryptanalysis, is the backbone of communications intelligence"

- Stewart Baker (NSA): "metadata **ABSOLUTELY TELLS YOU EVERYTHING ABOUT SOMEBODY'S LIFE**. If you have enough metadata, you don't really need content."

- Tempora, MUSCULAR → XkeyScore, PRISM

- Also "good" uses: recommendations, location-based services,

Herman, Michael. Intelligence power in peace and war. Cambridge University Press, 1996.

Diffie, Whitfield, and Susan Landau. Privacy on the line: The politics of wiretapping and encryption. MIT press, 2010.

<http://www.theguardian.com/world/interactive/2013/nov/01/snowden-nsa-files-surveillance-revelations-decoded>

# WE NEED TO PROTECT THE COMMUNICATION LAYER!

## ANONYMOUS COMMUNICATIONS

### ➤ GENERAL APPLICATIONS

- Freedom of speech
- Profiling / price discrimination
- Spam avoidance
- Investigation / market research
- Censorship resistance

### ➤ SPECIALIZED APPLICATIONS

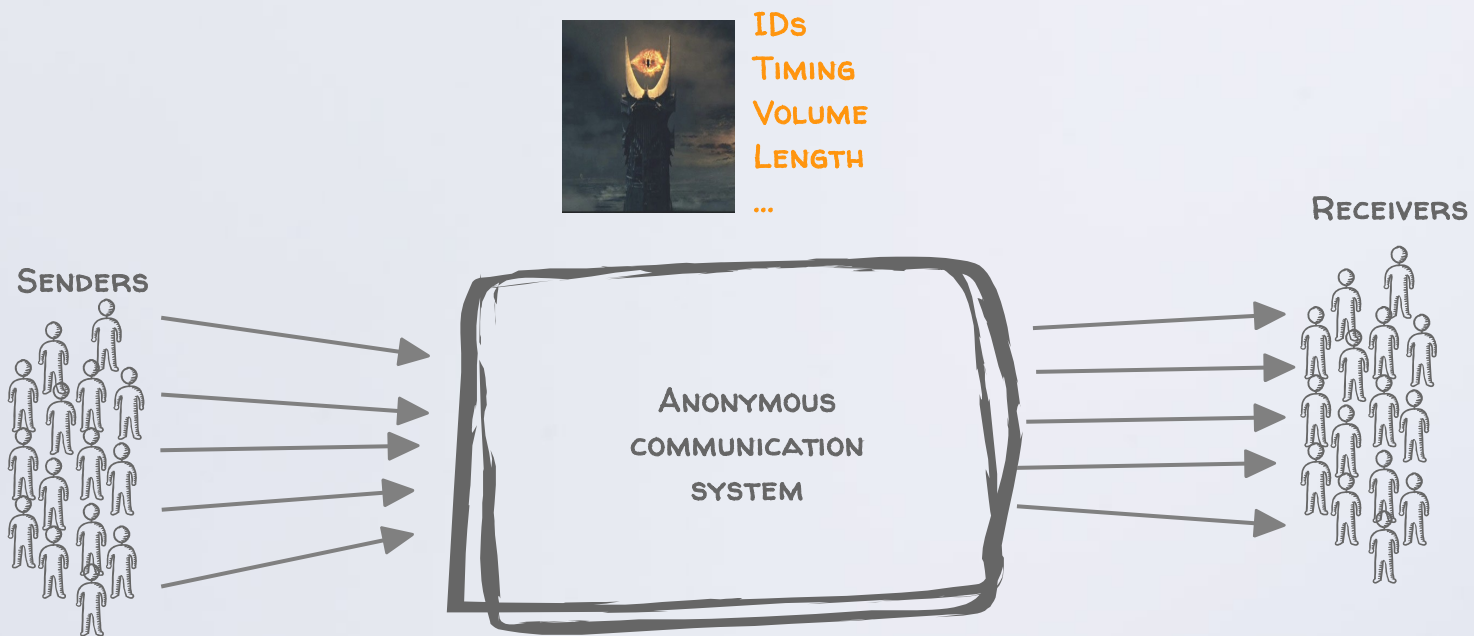
- Electronic voting
- Auctions / bidding / stock market
- Incident reporting
- Witness protection / whistle blowing
- Showing anonymous credentials!

#### Anonymity is important to:

- the people who run some of the funniest parody Twitter accounts, such as [@FeministHulk](#) (SMASH THE PATRIARCHY!) or [@BPGlobalPr](#) during the Deepwater Horizon aftermath. San Francisco would not be better off if we knew who was behind [@KarltheFog](#), the most charming personification of a major city's climate phenomenon.
- the young LGBTQ youth seeking advice online about coming out to their parents.
- the marijuana grower who needs to ask questions on an online message board about lamps and fertilizer or complying with state law, without publicly admitting to committing a federal offense.
- the medical patient seeking advice from other patients in coping with a chronic disease, whether it's alopecia, irritable bowel syndrome, cancer or a sexually transmitted infection.
- the online dater, who wants to meet new people but only reveal her identities after she's determined that potential dates are not creeps.
- the business that wants no-pulled-punches feedback from its customers.
- the World of Warcraft player, or any other MMOG gamer, who only wants to engage with other players in character.
- artists. Anonymity is integral to the work of The Yes Men, Banksy and Keizer.
- the low-income neighborhood resident who wants to comment on an article about gang violence in her community, without incurring retribution in the form of spray paint and broken windows.
- the boyfriend who doesn't want his girlfriend to know he's posing questions on a forum about how to pick out a wedding ring and propose. On the other end: Anonymity is important to anyone seeking advice about divorce attorneys online.
- the youth from an orthodox religion who secretly posts reviews on hip hop albums or R-rated movies.
- the young, pregnant woman who is seeking out advice on reproductive health services.
- the person seeking mental health support from an online community. There's a reason that support groups so often end their names with "Anonymous."
- the job seeker, in pursuit of cover letter and resume advice in a business blogger's comments, who doesn't want his current employer to know he is looking for work.
- many people's sexual lives, whether they're discussing online erotica or arranging kink meet-ups.
- Political Gabfest listeners. Each week, the hosts encourage listeners to post comments. Of the 262 largely positive customer reviews on iTunes, only a handful see value in using their real names.

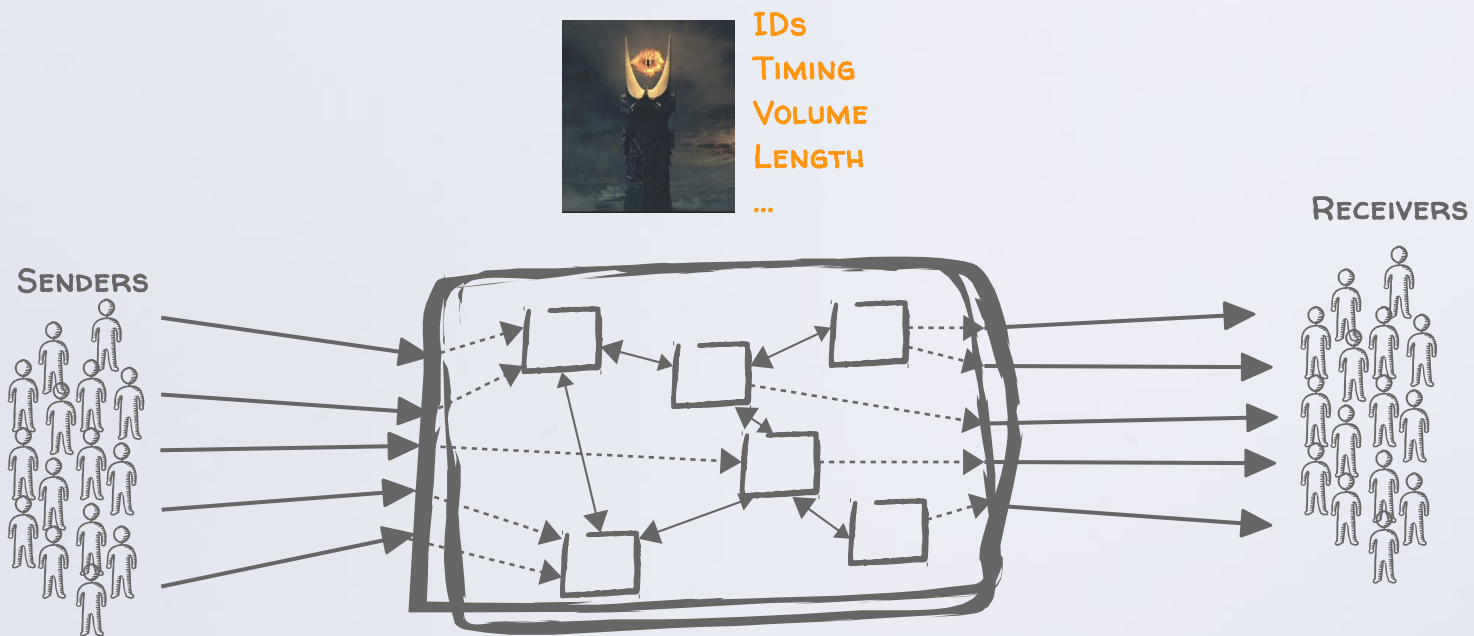
<https://www.eff.org/deeplinks/2013/10/online-anonymity-not-only-trolls-and-political-dissidents>  
[http://geekfeminism.wikia.com/wiki/Who\\_is\\_harmed\\_by\\_a\\_%22Real\\_Names%22\\_policy%3F](http://geekfeminism.wikia.com/wiki/Who_is_harmed_by_a_%22Real_Names%22_policy%3F)

# ANONYMOUS COMMUNICATIONS: ABSTRACT MODEL



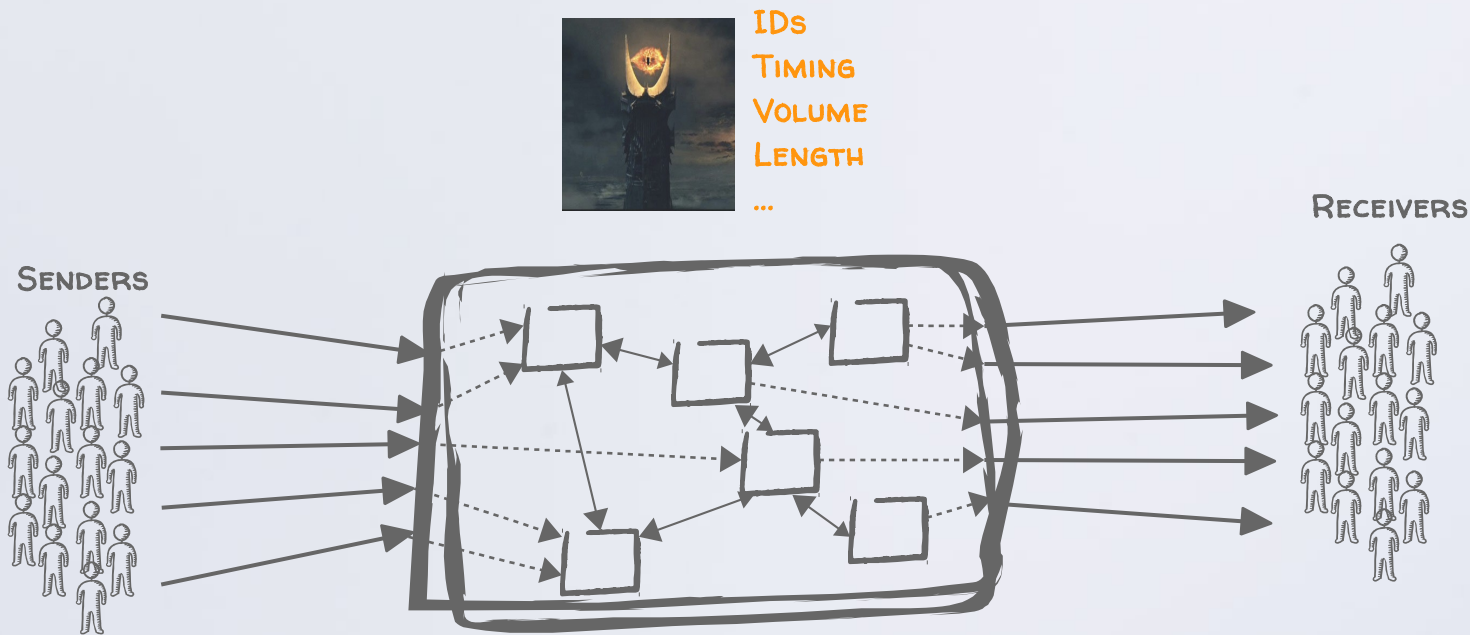
- **BITWISE UNLINKABILITY**
  - Crypto to make inputs and outputs bit patterns different
- **(RE)PACKETIZING + (RE)SCHEDULE**
  - Destroy patterns (traffic analysis resistance)

# ANONYMOUS COMMUNICATIONS: ABSTRACT MODEL



- **BITWISE UNLINKABILITY**
  - Crypto to make inputs and outputs bit patterns different
- **(RE)PACKETIZING + (RE)SCHEDULE + (RE)ROUTING,**
  - Destroy patterns (traffic analysis resistance)
  - Load balancing
  - Distribute trust

# IN THEORY SHOULD WORK, BUT IN PRACTICE...



➤ **BITWISE UNLINKABILITY**

- Crypto to make inputs and outputs bit patterns different



➤ **(RE)PACKETIZING + (RE)SCHEDULE + (RE)ROUTING,**

- Destroy patterns (traffic analysis resistance)
- Load balancing
- Distribute trust

Bandwidth

Delay

Churn



Intrinsic network differences

Trust?

# ... STILL VULNERABLE TO TRAFFIC ANALYSIS

## FIND PROFILES AND COMMUNICATION PATTERNS

persistent relationships show up

## DEVICE IDENTIFICATION / LOCATION

hosts' hardware particular characteristics

## TRACE TRAFFIC BASED ON PATTERNS

number of packets, delays, ... differ per flow

## IDENTIFY USERS BASED ON CHOICES

not everybody can choose everything

## IDENTIFY TRAFFIC BASED ON THEIR PATTERNS

(E.G., WEBSITE FINGERPRINTING)

same traffic always looks similar

## RECOVER CONTENT

timing and length of packets

## TRACE PACKETS BASED ON ROUTING ALGORITHMS

not all routes are possible

## USERS' PAST HISTORY

timing correlated to caches

**MANY, MANY, MANY, MANY, MANY MORE....**

Pérez-González, Fernando, and Carmela Troncoso. "Understanding statistical disclosure: A least squares approach." PETS, 2012.

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hosts' hardware particular characteristics

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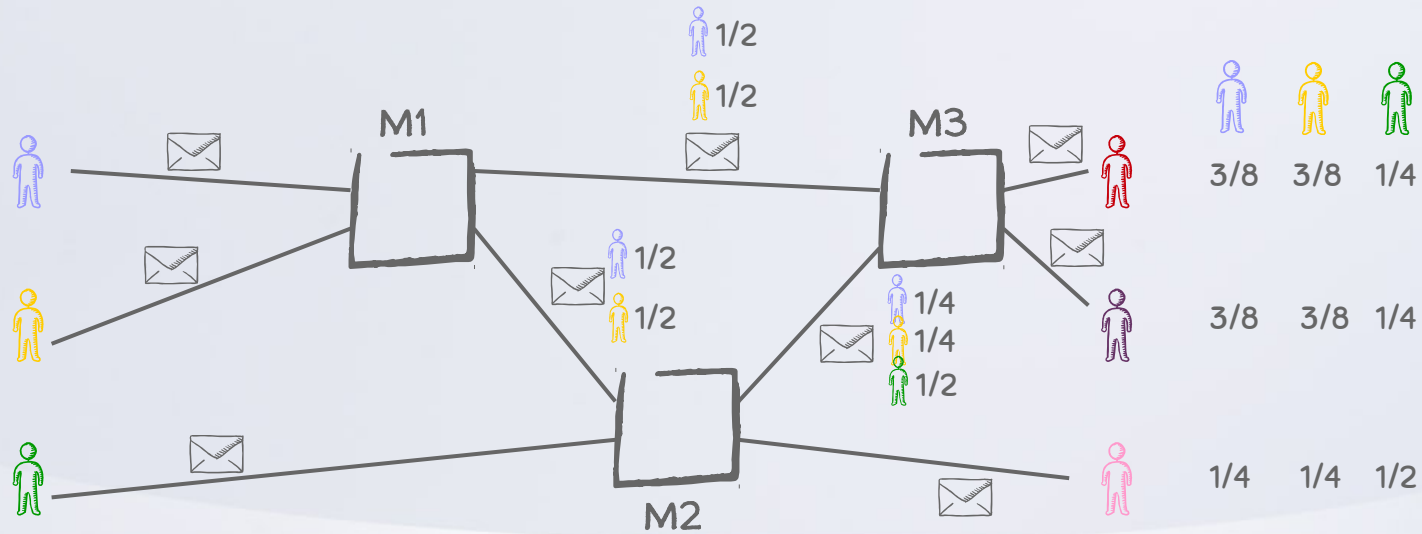
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# WHERE DO MESSAGES GO?

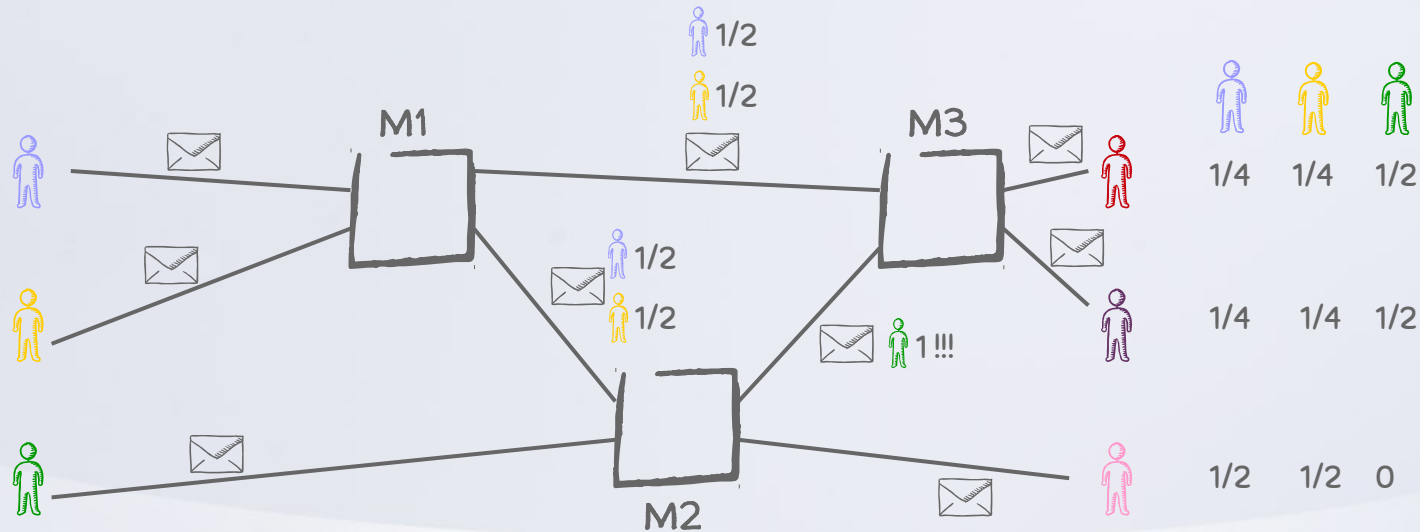
THRESHOLD MIX: collects  $t$  messages, and outputs them changing their appearance and in a random order



# WHERE DO MESSAGES GO?

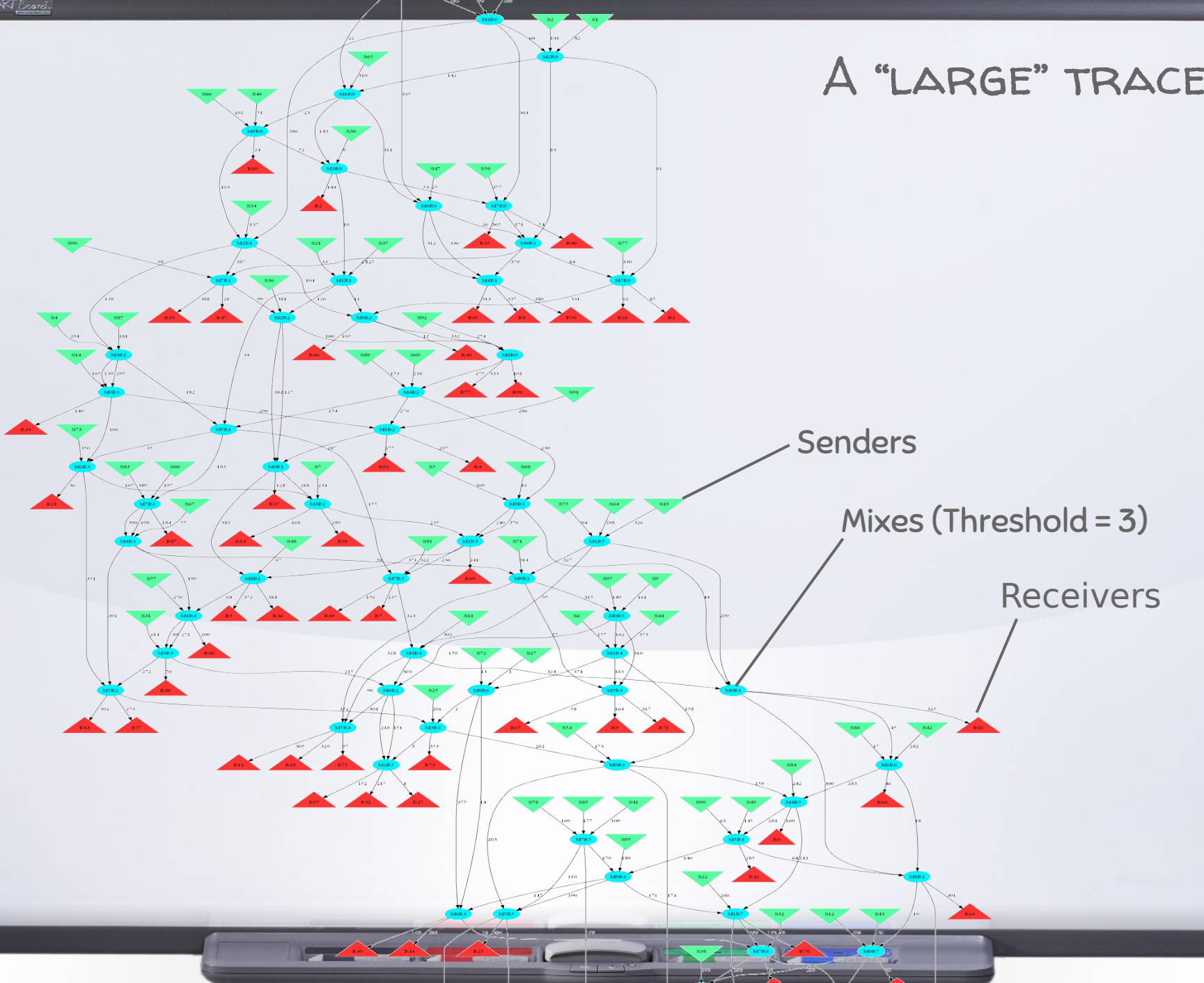
not everything is possible (e.g., max 2 hops)

THRESHOLD MIX: collects  $t$  messages, and outputs them changing their appearance and in a random order



**NON TRIVIAL GIVEN  
OBSERVATION!!**

# A "LARGE" TRACE



Senders

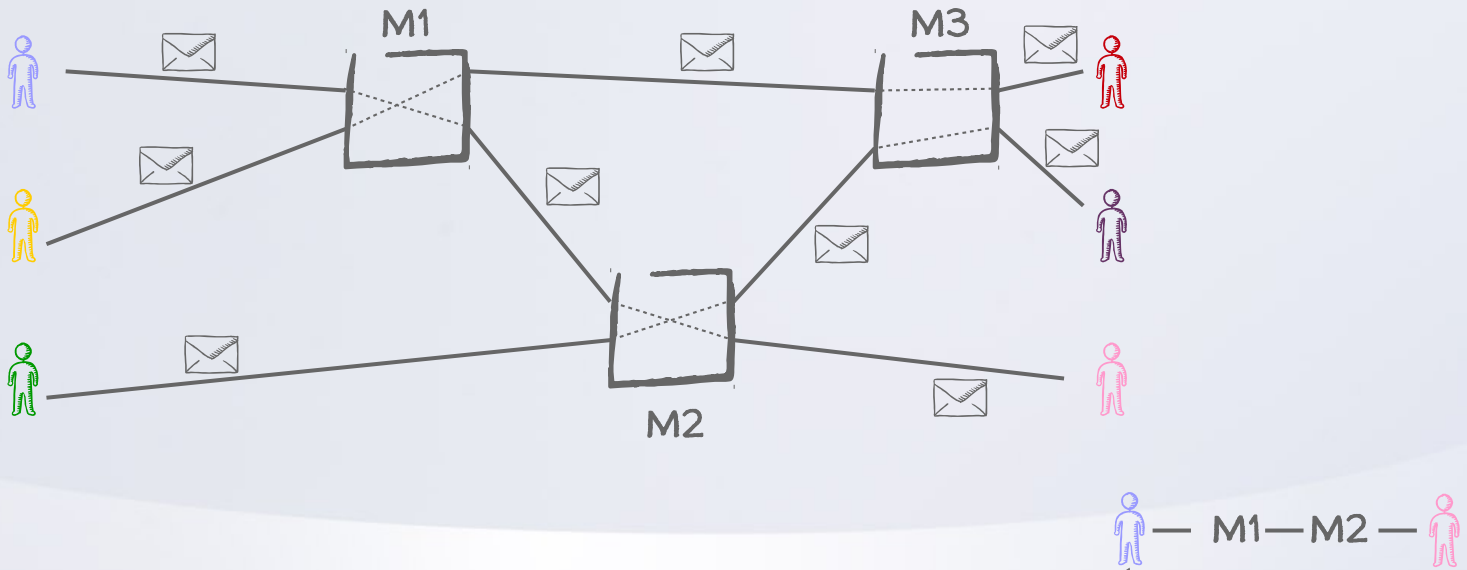
Mixes (Threshold = 3)

Receivers

# REDEFINING THE PROBLEM

Given what we see (OBSERVATION) and the system operation (CONSTRAINTS)

Probability of mixes "HIDDEN STATE"?  
(or Probability of each possible path?)



$$Pr[HS|O,C] = \frac{Pr[O|HS,C] \cdot Pr[HS|C]}{\sum_{HS} Pr[HS,O|C]} = \frac{Pr[O|HS,C] \cdot K}{Z} = \frac{Pr[Paths|C] \cdot K}{Z}$$

# ACTUALLY...

We usually care about marginal probabilities, not all ( $\Pr[\text{blue} \rightarrow \text{red} | O, C]$ )



$$\Pr[A \rightarrow B | O, C] = \sum_{HS} I(A \rightarrow B \in HS) \cdot \Pr[HS | O, C]$$



But we could also compute them using samples. If we had:

$$HS_1, HS_2, HS_3, \dots, HS_N \sim \Pr[HS | O, C]$$

Simply count:

$$\Pr[A \rightarrow B | O, C] = \frac{\sum_{HS} I(A \rightarrow B \in HS)}{N}$$

MCMC

$$\Pr[HS | O, C] = \frac{\Pr[Paths | C] \cdot K}{Z}$$

$$\prod_{\text{senders}} \Pr[Path | C]$$

Example: in Tor a path is one guard, one middle, one exit chosen with respect to a known algorithm "proportionally" to their bandwidth

# TAKEAWAYS ATTACKS ON ROUTES

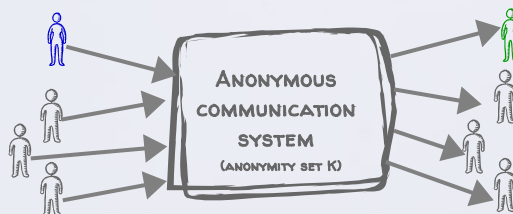
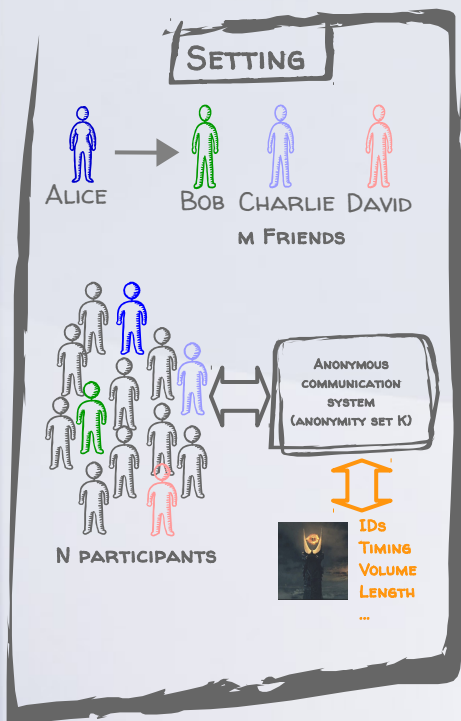
- Traffic analysis is non trivial when there are constraints
- Traffic analysis as inference problem: systematic!
  - Probabilistic model: can incorporate most attacks
    - Can integrate knowledge on path probability computation
      - More constraints → less anonymity but more complexity
    - Combines well with other inferences: e.g., long-term attacks (in a minute)
- MCMC methods to extract marginal probabilities
  - Systematic
  - Only generative model needed


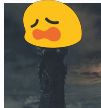
# FINDING PERSISTENT COMMUNICATIONS DISCLOSURE ATTACKS

IN REALITY...

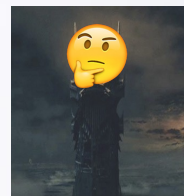
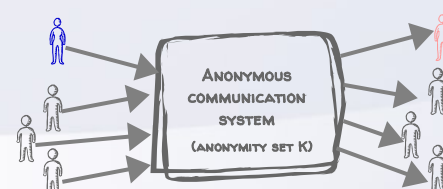
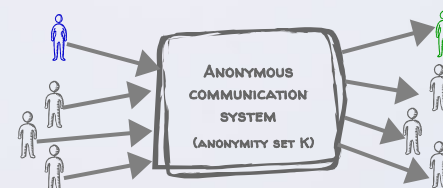
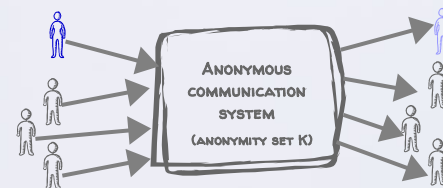
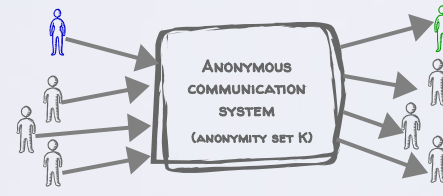
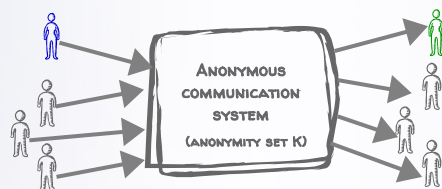
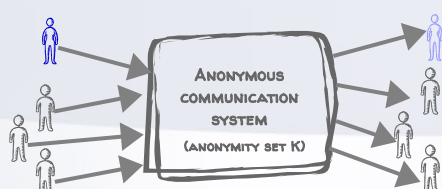
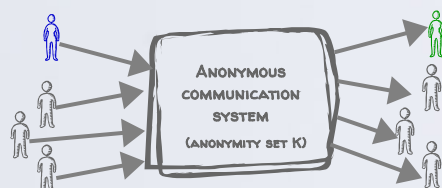
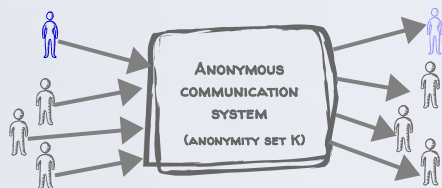
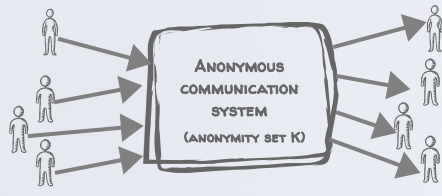
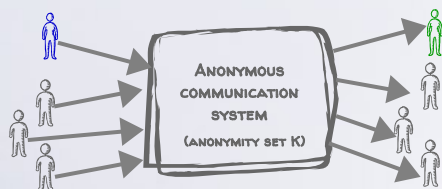
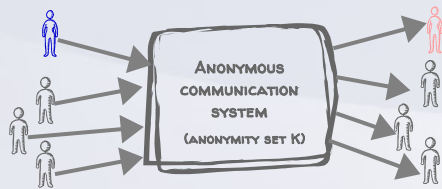
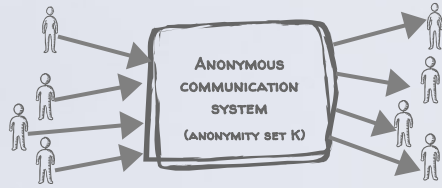
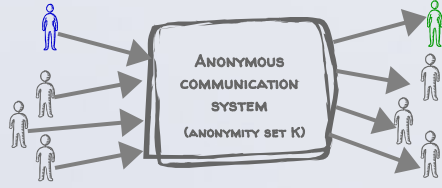
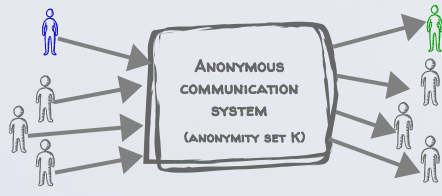
ALICE HAS FEW FRIENDS WITH WHOM SHE COMMUNICATES OFTEN  
ALICE IS NOT ALWAYS ONLINE (AT LEAST NOT ACTIVE)

CAN SAURON LEARN ALICE'S FRIENDS?



- 1-  SEES ALICE SENDING A SINGLE MESSAGE TO THE SYSTEM
- 2- ANONYMITY SET SIZE = K
- 3- PERFECT! 

# AS TIME GOES BY AND ALICE SENDS MORE MESSAGES...



-  x 8
-  x 2
-  x 3



# LET'S "DO" THE MATH

N=20 M=3 K=5 T=45

ALICE'S FRIENDS={[0, 13, 19]}

## APPROACH 1: STATISTICAL DISCLOSURE ATTACK

- > Alice's friends will be in the sets more often than random receivers. How often?

Expected number of messages per receiver after t rounds:

- >  $\mu_{\text{other}} = (1 / N) \cdot (K-1) \cdot t$

- >  $\mu_{\text{Alice}} = (1 / M) \cdot t + \mu_{\text{other}}$

- > Just count the number of messages per receiver when Alice is sending!

- >  $\mu_{\text{Alice}} > \mu_{\text{other}}$

### Round Receivers

Round	Receivers	SDA
1	[15, 13, 14, 5, 9]	[13, 14, 15]
2	[19, 10, 17, 13, 8]	[13, 17, 19]
3	[0, 7, 0, 13, 5]	[0, 5, 13]
4	[16, 18, 6, 13, 10]	[5, 10, 13]
5	[1, 17, 1, 13, 6]	[10, 13, 17]
6	[18, 15, 17, 13, 17]	[13, 17, 18]
7	[0, 13, 11, 8, 4]	[0, 13, 17]
8	[15, 18, 0, 8, 12]	[0, 13, 17]
9	[15, 18, 15, 19, 14]	[13, 15, 18]
10	[0, 12, 4, 2, 8]	[0, 13, 15]
11	[9, 13, 14, 19, 15]	[0, 13, 15]
12	[13, 6, 2, 16, 0]	[0, 13, 15]
13	[1, 0, 3, 5, 1]	[0, 13, 15]
14	[17, 10, 14, 11, 19]	[0, 13, 15]
15	[12, 14, 17, 13, 0]	[0, 13, 17]
16	[18, 19, 19, 8, 11]	[0, 13, 19]
17	[4, 1, 19, 0, 19]	[0, 13, 19]
18	[0, 6, 1, 18, 3]	[0, 13, 19]
19	[5, 1, 14, 0, 5]	[0, 13, 19]
20	[17, 18, 2, 4, 13]	[0, 13, 19]
21	[8, 10, 1, 18, 13]	[0, 13, 19]
22	[14, 4, 13, 12, 4]	[0, 13, 19]
23	[19, 13, 3, 17, 12]	[0, 13, 19]
24	[8, 18, 0, 10, 18]	[0, 13, 18]

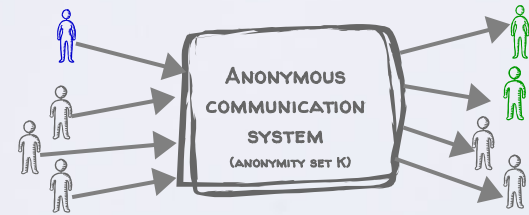
Danezis, George. "Statistical disclosure attacks." Security and Privacy in the Age of Uncertainty, 2003.

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Mathewson, Nick, and Roger Dingledine. "Practical traffic analysis: Extending and resisting statistical disclosure." PETS, 2004

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# LET'S "DO" THE MATH



$P_{ij}$  = probability that  $i$  sends a message to  $j$   
 $x^r$  = vector of  $n$ # of messages sent round  $r$  ( $x_i^r = 1$ )  
 $y^r$  = vector of  $n$ # of messages received round  $r$  ( $y_j^r = 2$ )  
 $H = [x^1, x^2, x^3, \dots]$

## APPROACH 2: LEAST SQUARES DISCLOSURE ATTACK

- Maximum likelihood approach: solve a Least Squares minimizing mean squared error between real and estimated profiles

$$\hat{p} = \underset{p}{\operatorname{arg\,min}} \|y - Hp\|$$

$$\begin{aligned}
 & p_{i,j} \leq 1 \\
 & \sum_i p_{i,j} = 1
 \end{aligned}
 \quad \Rightarrow \quad
 \hat{p} = (H^T H)^{-1} H^T y$$

- Analytical expressions that describe the evolution of the profiling error

$$\text{MSE} = \|p - \hat{p}\|^2 = \frac{1}{t} \left( N - 1 + \frac{1}{k} \right) \left( N - \sum_j \frac{f_j^2}{f^2 N} \right)$$

Diagram annotations:
 

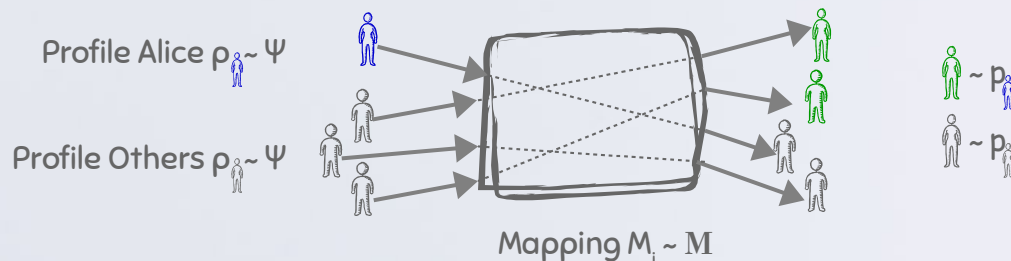
- Upward arrow from  $\frac{1}{t}$  to "Users"
- Downward arrow from  $\frac{1}{t}$  to "rounds"
- Downward arrow from  $\frac{1}{k}$  to "Batch size"
- Downward arrow from  $\frac{f_j^2}{f^2 N}$  to "Senders that send a lot"
- Upward arrow from  $\frac{f_j^2}{f^2 N}$  to "Receivers receive from many"

Pérez-González, Fernando, and Carmela Troncoso. "Understanding statistical disclosure: A least squares approach." PETS, 2012.

Oya, Simon, Carmela Troncoso, and Fernando Pérez-González. "Do dummies pay off? limits of dummy traffic protection in anonymous communications." PETS, 2014

Perez-Gonzalez, Fernando, Carmela Troncoso, and Simon Oya. "A least squares approach to the static traffic analysis of high-latency anonymous communication systems." TIFS 2014

# LET'S "DO" THE MATH



## APPROACH 3: DISCLOSURE ATTACK AS AN INFERENCE PROBLEM

- > What we are looking for:  $\Pr[p_A, p_O, M_i | O, M, \Psi]$
- > More concretely, marginal probabilities & distributions
  - >  $\Pr[\text{Alice} \rightarrow \text{Bob}]$  – Are Alice and Bob friends?
  - >  $M_x$  – Who is talking to whom at round  $x$ ?
  - > Solve through sampling!

Profiles:  $\Pr[p_A, p_O | M_i, O, M, \Psi, K]$   
 (Direct sampling by sampling Dirichlet dist.)

Mappings:  $\Pr[M_i | p_A, p_O, O, M, \Psi, K]$   
 (Direct sampling of the matching link by link)

## GIBBS SAMPLING

- > Allows sampling from complex distributions when their marginal distributions are easy to sample from.
- > Example: Sample  $\Pr[A, B | O]$
- > For sample  $s$  in  $(0, \text{SAMPLES})$ :
  - > For iteration  $j$  in  $(0, \text{ITERATIONS})$ :
    - >  $a_j \sim A$  with  $\Pr[A|B=b_{j-1}, O]$
    - >  $b_j \sim B$  with  $\Pr[B|A=a_j, O]$
  - >  $\text{Sample}_s = (a_{\text{SAMPLES}}, b_{\text{SAMPLES}})$

# PERSISTENT PATTERNS TAKEAWAYS

- Near-perfect anonymity is not perfect enough!
  - High level patterns cannot be hidden for ever
  - Unobservability / maximal anonymity is needed
- Three approaches to the problem (actually I skipped the seminal work)

## SDA

- Simple
- Fast!
- Best result not guaranteed
  - Only that one

## LSDA

- Flexible
- Fast!
- Optimal result (MSE)
  - But only that one
- Error prediction
- Design tool!

## INFERENCE

- Flexible
- “expensive”
- Distribution
  - Many quantities
  - Confidence intervals
- Not best solution

# ARE WE DOOMED?

- Countermeasures
  - Delay: plain batching does not seem the best
    - Pool mixes
    - Attacks can be adapted to account for more complex delay patterns
  - Dummy traffic: include “fake packets” to disorient the adversary
    - How do we make them indistinguishable?
    - Who decides about them?
  - This is GPA, other adversary models?
    - Actually Tor has other goal!

# SUMMARY

- › The Lord of The Rings is a great timeless book
- › Crypto protects data, but does not always protect privacy
- › Traffic analysis is the art of exploiting meta-data to extract information
- › Traffic analysis can exploit a gazillion features: protecting efficiently is difficult!
  - › Recovering persistent patterns, tracing messages in restricted routes
- › Different attack flavors provide different trade-offs

# CHALLENGES

- Countermeasures! Dummies? Delays? Efficient combination
  - Systematic design?
- Privacy metric, what is the goal?
- Modeling adversarial knowledge
- Other fields... location privacy, behavioral/contextual authentication

# THANKS!

## ANY QUESTIONS?

More about traffic analysis: <https://www.petsymposium.org/>



carmela.troncoso@imdea.org  
<https://software.imdea.org/~carmela.troncoso/>  
(these slides will be there soon)

Template: <http://www.brainybetty.com/>  
Figures: [SlidesCarnival](#)



# LET'S "DO" THE MATH

N=20 m=3 K=5 t=45  
 Alice's Friends={[0, 13, 19]}

## APPROACH O: (HITTING SET) DISCLOSURE ATTACK

- > Idea: "the only people that are in the intersection of all Alice's rounds are her friends"
- > Guess the set of friends of Alice:
  - > Constraint  $|R_A| = m$
  - > Accept if an element is in the output of each round
- > Downside: Cost
  - > N receivers, m size - (N choose m) options
  - > Exponential → Bad [good approximations exist]
- > Comparison:
  - > Computationally very expensive
  - > Limited model
  - > Difficult to apply to complex systems

Round	Receivers	SDA	HS
<b>1</b>	<b>[15, 13, 14, 5, 9]</b>	<b>[13, 14, 15]</b>	<b>685</b>
2	[19, 10, 17, 13, 8]	[13, 17, 19]	395
3	[0, 7, 0, 13, 5]	[0, 5, 13]	257
4	[16, 18, 6, 13, 10]	[5, 10, 13]	203
5	[1, 17, 1, 13, 6]	[10, 13, 17]	179
6	[18, 15, 17, 13, 17]	[13, 17, 18]	175
7	[0, 13, 11, 8, 4]	[0, 13, 17]	171
8	[15, 18, 0, 8, 12]	[0, 13, 17]	80
9	[15, 18, 15, 19, 14]	[13, 15, 18]	41
10	[0, 12, 4, 2, 8]	[0, 13, 15]	16
11	[9, 13, 14, 19, 15]	[0, 13, 15]	16
12	[13, 6, 2, 16, 0]	[0, 13, 15]	16
13	[1, 0, 3, 5, 1]	[0, 13, 15]	4
14	[17, 10, 14, 11, 19]	[0, 13, 15]	2
15	[12, 14, 17, 13, 0]	[0, 13, 17]	2
<b>16</b>	<b>[18, 19, 19, 8, 11]</b>	<b>[0, 13, 19]</b>	<b>1</b>
17	[4, 1, 19, 0, 19]	[0, 13, 19]	1
18	[0, 6, 1, 18, 3]	[0, 13, 19]	1
19	[5, 1, 14, 0, 5]	[0, 13, 19]	1
20	[17, 18, 2, 4, 13]	[0, 13, 19]	1
21	[8, 10, 1, 18, 13]	[0, 13, 19]	1
22	[14, 4, 13, 12, 4]	[0, 13, 19]	1
23	[19, 13, 3, 17, 12]	[0, 13, 19]	1
24	[8, 18, 0, 10, 18]	[0, 13, 18]	1

